



Natural convection from a confined horizontal cylinder: the optimum distance between the confining walls

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Received 27 May 1999; received in revised form 15 March 2000

Abstract

The laminar natural convection from an isothermal horizontal cylinder confined between vertical walls, at low Rayleigh numbers, is investigated by theoretical, experimental and numerical methods. The height of the walls is kept constant, however, their distance is changed to study its effect on the rate of the heat transfer. Results are incorporated into a single equation which gives the Nusselt number as a function of the ratio of the wall distance to cylinder diameter, t/D , and the Rayleigh number. There is an optimum distance between the walls for which heat transfer is maximum. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

One of the objectives of the investigation about natural convection in the recent years has been to explore the applications of the electric cooling. Natural convection from different geometries are studied and techniques are developed to enhance the rate of heat transfer.

One of the geometries which has received considerable attention has been an isothermal horizontal cylinder confined between vertical walls.

Marsters [1] was the first to address this problem systematically, using both experimental and analytical methods. His experimental results cover a vast range of Rayleigh numbers. He studied the effects of changes in the height and spacing of the walls, on the Nusselt

number. He did not observe any optimum wall spacing for the maximum Nusselt number.

The second investigation was the numerical work of Gucceri and Farouk [2]. They used the finite difference method to solve the energy and momentum equations in the stream vorticity form. They observed a mixed heat transfer behavior from the cylinder, regarding the effects of the wall spacing, depending on the values of the Rayleigh number.

Pfeil and Sparrow [3] studied the problem experimentally. They generated fifteen sets of data by changing the wall height and spacing. They observed that the rate of heat transfer from the cylinder increased with reduction in the distance between the walls. This effect was more noticeable for the smaller Rayleigh numbers. They did not observe any optimum wall spacing, though.

Karim et al. [4] studied the problem, experimentally, and presented a correlation for the Nusselt number. They did not observe any optimum wall spacing, either.

Sadeghipour and Kazemzadeh [5] investigated the transient natural convection from a confined isother-

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Nomenclature

A	heat transfer area of cylinder	u, v	components of fluid velocity
C_D	drag coefficient for cylinder	x, y	space variables measured from inlet to the wall region (nondimensionalized with respect to H and t)
C_p	thermal capacitance	x', y'	space variables measured from center of cylinder (nondimensionalized with respect to D)
C_1	coefficient in momentum equation		
C_1'	modified C_1 coefficient		
C_2	coefficient in energy equation		
C_2'	modified C_2 coefficient		
C_3	coefficient of dissipation loss in energy equation		
D	diameter of cylinder	<i>Greek symbols</i>	
f	friction factor	β	volumetric expansion coefficient
F	radiation shape factor	ε	emissivity
g	acceleration due to gravity	η	modified wall spacing to cylinder diameter ratio
Gr_D	($= g\beta\Delta T_s D^3/\nu^2$), Grashof number	ν	kinematic viscosity
H	height of adiabatic walls	ρ	density
h	heat transfer coefficient	σ	Stephan–Boltzman constant
k	thermal conductivity of air	τ	shear stress on the wall
L	length of cylinder		
\dot{m}	mass flow rate	<i>Subscripts</i>	
Nu	Nusselt number	1	inlet condition
\overline{Nu}	modified Nusselt number	2	outlet condition
p	pressure	∞	ambient
Pr	Prandtl number	w	wall condition
\dot{Q}	total rate of heat transfer from cylinder	s	surface of cylinder
Q_{conv}	rate of heat transfer from the cylinder by convection		
Ra_D	Rayleigh number	<i>Superscripts</i>	
Re_D	Reynolds number	$\hat{\quad}$	dimensionless quantities, with respect to H and t
t	wall spacing	\sim	dimensionless quantities, with respect to D
T	temperature	*	reference velocity
u^*	($= \alpha/D$), reference velocity		

mal cylinder, numerically. They solved their problem for one Rayleigh number ($Ra = 1000$), and presented the variation of Nusselt number with time. They observed an optimum wall distance to cylinder diameter ratio for the maximum heat transfer.

Finally, Ma et al. [6] studied the effects of variation in the vertical location of the cylinder on its rate of heat transfer, numerically. They too, observed mixed behavior.

In this paper, we are presenting the results of our theoretical, numerical and experimental investigations of the steady state natural convection from an isothermal cylinder confined between two adiabatic vertical walls, for low Rayleigh numbers ($Ra = 650$ – 1000). The height of the walls is kept constant ($H/D = 7$), and the effects of the changes in the wall spacing on the rate of heat transfer are determined.

The limiting cases of $t/D \rightarrow 1$ and $t/D \rightarrow \infty$ are

addressed in the analytical section. Then, sets of the numerical and experimental data are generated for the rate of heat transfer for different values of t/D and Ra .

Results are incorporated into a single correlation which gives the variation of Nu with t/D and Ra for $H/D = 7$.

2. Problem description

The problem under investigation is heat transfer from horizontal isothermal cylinder confined between two vertical adiabatic walls. Fig. 1 shows the geometry and the configuration of the problem. Problem is assumed to be two-dimensional, with no variation of the conditions along the cylinder. The parameters which are kept constant during the investigation are the cylinder's diameter, D , the height of the walls, H ,

and the vertical location of the cylinder. The variable quantities have been the distance between the walls, t , and the cylinder's surface temperature, T_s .

3. Analytical solution

In this investigation, the Marsters [1] integral method was used to develop an analytical solution for the heat transfer behavior of the confined cylinder for the two extremes of $t/D \rightarrow 1$ and $t/D \rightarrow \infty$. Then the idea of intersection of asymptotes was utilized to show the existence of an optimum spacing for maximum rate of heat transfer. This technique has been put forward and has been successfully used for both natural and forced convection by Bejan [7–10] and used by others [11].

The governing continuity, momentum and energy equations are written first. For continuity of the flow between the inlet and outlet Section 1 and Section 2, we have;

$$\rho_1 u_1 t = \int_{-t/2}^{t/2} \rho_2 u_2 dy$$

and by changing the variable y as;

$$\hat{y} = \frac{y}{t}$$

the continuity equation is written as;

$$\dot{m} = \rho_1 u_1 t = t \int_{1/2}^{-1/2} \rho_2 u_2 d\hat{y} \tag{1}$$

The momentum equation, a balance between the buoyancy force, the chimney effects and the friction forces on the confining walls and on the cylinder, with the momentum changes, is written as;

$$\begin{aligned} (p_1 - p_2)t - \int_0^H \tau dx - C_D \frac{1}{2} \rho_1 u_1^2 D \\ - g \rho_1 \int_0^H \int_{-t/2}^{t/2} \frac{\rho}{\rho_1} dy dx \\ = \int_{-t/2}^{t/2} \rho_2 u_2^2 dy - \dot{m} u_1 \end{aligned} \tag{2}$$

On the other hand, the inlet and outlet pressures can be written as;

$$p_1 = p_\infty - \frac{1}{2} \rho_1 u_1^2 \tag{3}$$

$$p_2 = p_\infty - \rho_1 g H \tag{4}$$

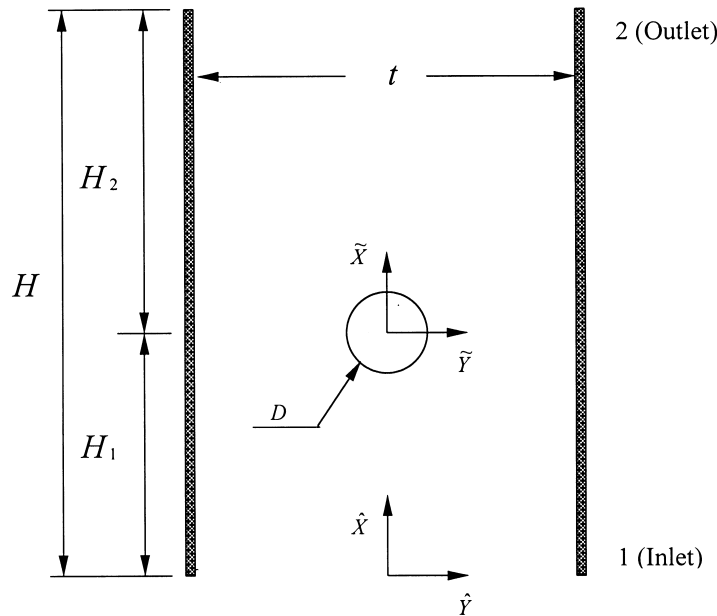


Fig. 1. Configuration of the problem and the coordinate systems ($H_1 = 3D$, $H_2 = 4D$).

Introducing Eqs. (3) and (4) and changing the variable x as $\hat{x} = x/H$, Eq. (2) can be written as;

$$\frac{gH}{u_1^2} \left(1 - \int_0^1 \int_{-1/2}^{1/2} \frac{\rho}{\rho_1} d\hat{y} d\hat{x} \right) - \left(\frac{1}{2} + \frac{1}{2} C_D \frac{D}{t} \right) - \frac{H}{t} \int_0^1 \frac{\tau(\hat{x}) d\hat{x}}{\rho_1 u_1^2} = \int_{-1/2}^{1/2} \frac{\rho_2 u_2^2}{\rho_1 u_1^2} d\hat{y} - 1 \quad (5)$$

Defining ρ as;

$$\rho = \rho_1(1 - \beta \Delta T)$$

where

$$\Delta T = T - T_\infty$$

In Eq. (5), the momentum equation can finally be written as

$$\frac{Gr_D}{Re_D^2} \cdot \frac{H}{D} \int_0^1 \int_{-1/2}^{1/2} \frac{\Delta T}{\Delta T_s} d\hat{x} d\hat{y} - \frac{1}{2} \left(1 + C_D \frac{D}{t} \right) - \frac{H}{t} \int_0^1 \frac{\tau}{\rho_1 u_1^2} d\hat{x} = \int_{-1/2}^{1/2} \left(\frac{\rho_2 u_2^2}{\rho_1 u_1^2} - 1 \right) d\hat{y} \quad (6a)$$

where

$$Re_D = \frac{u_1 D}{\nu} \quad (6b)$$

$$Gr_D = \frac{g\beta\Delta T_s D^3}{\nu^2}$$

The energy equation expresses a balance between the heat transfer from the cylinder and changes in the flow energy between the inlet and the outlet. This equation is, then, written as;

$$\begin{aligned} \dot{Q} &= tC_p \int_{-1/2}^{1/2} \rho_2 u_2 T_2 d\hat{y} - tC_p \int_{-1/2}^{1/2} \rho_1 u_1 T_1 d\hat{y} \\ &+ \frac{t}{2} \int_{-1/2}^{1/2} \rho_2 u_2^3 d\hat{y} - \frac{t}{2} \int_{-1/2}^{1/2} \rho_1 u_1^3 d\hat{y} \\ &+ gt \int_{-1/2}^{1/2} \rho_2 u_2 x_2 d\hat{y} - gt \int_{-1/2}^{1/2} \rho_1 u_1 x_1 d\hat{y} \end{aligned} \quad (7)$$

Since $x_2 - x_1 = H$, Eq. (7) can be rearranged as;

$$\begin{aligned} \frac{Nu}{Pr Re_D} \left(\frac{D}{t} \right) &= \frac{T_1}{\Delta T_s} \int_{-1/2}^{1/2} \left(\frac{\rho_2 u_2 T_2}{\rho_1 u_1 T_1} - 1 \right) d\hat{y} \\ &+ \frac{u_1^2}{2C_p \Delta T_s} \int_{-1/2}^{1/2} \left(\frac{\rho_2 u_2^3}{\rho_1 u_1^3} - 1 \right) d\hat{y} + \frac{gH}{C_p \Delta T_s} \end{aligned} \quad (8)$$

where;

$$\dot{Q} = \pi Dh \Delta T_s$$

and

$$Nu = hD/k$$

Two extreme cases are considered for this problem which are $t/D \rightarrow 1$ and $t/D \rightarrow \infty$. Now, we will simplify the governing equations for these two cases.

a — The limit $t/D \rightarrow \infty$

As the distance between confining walls is increased, their effect on the rate of heat transfer from the cylinder vanishes, gradually. For $t/D \rightarrow \infty$ the solution of this problem should eventually approach that of heat transfer from a single cylinder with no confining walls.

In this case, neglecting the inertia, the buoyancy force should balance the friction force on the cylinder,

$$C_1 \frac{Gr_D}{Re_D^2} \cdot \frac{H}{D} \approx \frac{1}{2} C_D \frac{D}{t} \quad (9)$$

where

$$C_1 = \int_0^1 \int_{-1/2}^{1/2} \frac{\Delta T}{\Delta T_s} d\hat{y} d\hat{x}$$

Choosing the proper value for C_D [12], Eq. (9) can be arranged as;

$$Gr_D = \frac{2.742}{C_1} Re_D^{1.75} \left(\frac{D}{t} \right) \left(\frac{D}{H} \right) \quad (10)$$

The Nusselt number for a single isothermal cylinder is given as [13];

$$Nu = 0.85 Ra^{0.188} \quad (11)$$

By neglecting the kinetic and potential energy effects in the energy equation (8), the Nusselt number may be obtained as;

$$Nu = \frac{t}{D} Re_D Pr \frac{T_1}{\Delta T_s} C_2 \quad (12)$$

The Nusselt number in Eq. (12) should approach a constant value for the limit $t/D \rightarrow \infty$. For this, we should have;

$$Re_D = \left(\frac{t}{D} \right)^{-1} \frac{1}{C_2 Pr T_1 / \Delta T_s} = \left(\frac{t}{D} \right)^{-1} \frac{1}{C_2'} \quad (13)$$

where

$$C_2 = \int_{-1/2}^{1/2} \left(\frac{\rho_2 u_2 T_2}{\rho_1 u_1 T_1} - 1 \right) d\hat{y} \quad (14a)$$

$$C_2' = C_2 Pr \frac{T_1}{\Delta T_s} \quad (14b)$$

Substituting Re_D from Eq. (13) into Eq. (10) gives;

$$Ra = Pr Gr_D = Pr \frac{2.742}{C_1 C_2^{1.75}} \left(\frac{t}{D}\right)^{-2.75} \left(\frac{D}{H}\right) \quad (15)$$

Substituting for Ra in Eq. (11) from Eq. (15), it becomes;

$$Nu = \frac{1.027}{C_1^{0.188} C_2^{0.329}} \cdot \frac{1}{\left(\frac{t}{D}\right)^{0.517} \left(\frac{H}{D}\right)^{0.188}} \quad (16)$$

where

$$C_1' = \frac{C_1}{Pr} \quad (17)$$

Eq. (17) shows that, for the limiting case of $t/D \rightarrow \infty$, the Nusselt number is inversely proportional to $(t/D)^{0.517}$. Therefore, the larger the value of t/D , the lower will be the Nusselt number.

b — The limit $t/D \rightarrow 1$

In this limit, if the confining walls are tall enough, neglecting the inertia, again, the buoyancy force will balance the friction force of the walls, therefore, we have;

$$\frac{Gr_D}{Re_D^2} \frac{H}{D} C_1 \approx \frac{H}{t} \int_0^1 \frac{f}{2} \frac{\rho u^2}{\rho_1 u_1^2} d\hat{x} \quad (18)$$

Let

$$f = \frac{24}{Re_{2t}} \quad (19)$$

After rearranging Eq. (18) we have;

$$Re_D \approx Gr_D \left(\frac{t}{D}\right)^2 \frac{C_1}{4C_3} \quad (20)$$

where

$$C_3 = \int_0^1 \frac{\rho u^2}{\rho_1 u_1^2} d\hat{x}$$

Substituting for Re_D in the energy equation (8) from Eq. (20) we have

$$Nu = \frac{C_1 C_4}{4C_3} \frac{T_1}{\Delta T_s} Ra_D \left(\frac{t}{D}\right)^3 \quad (21)$$

What is obvious from Eq. (21) is the high dependence of the Nusselt number on t/D . Nusselt number increases as the wall spacing increases. Eq. (21) is very much similar to what is given by Bejan et al. [10].

3.1. The optimum wall distance

Looking at the results obtained for the two cases ‘a’ and ‘b’, Eqs. (16) and (21), we observe that for case ‘a’, Nu decreases as t/D increases. However, for the case ‘b’, Nu increases with t/D . Therefore, the results for these two limiting cases intersect at a point where the rate of heat transfer from the cylinder is maximum.

Then a relation for the optimum t/D could be obtained as;

$$\left(\frac{t}{D}\right)_{opt} = \left(\frac{4.108 C_3}{C_1^{1.188} C_2^{1.329}}\right)^{0.285} \times \frac{1}{Pr^{0.04} (H/D)^{0.05}} \left(\frac{\Delta T_s}{T_1}\right)^{0.285} \frac{1}{Ra^{0.285}} \quad (22)$$

Eq. (22) shows that the $(t/D)_{opt}$ decreases as Ra increases. This probably was the reason why in some of the investigations at high Rayleigh numbers no optimum wall spacing was observed [1–4].

The interesting observation is the independent appearance of the Prandtl number in Eq. (22).

4. Numerical solution

4.1. The governing equations

The governing equations for free convection heat transfer from the cylinder of Fig. 1, in the form of differential equations, are given as

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \quad (23a)$$

$$\tilde{u} \frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + Pr \nabla^2 \tilde{v} \quad (23b)$$

$$\tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + Pr \nabla^2 \tilde{u} + Bo \tilde{T} \quad (23c)$$

$$\tilde{u} \frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{T}}{\partial \tilde{y}} = \nabla^2 \tilde{T} \quad (23d)$$

where the Boussinesq approximation is used for the buoyancy term in the vertical component of the momentum equation, and the dimensionless par-

ameters are defined as;

$$\tilde{p} = \frac{p}{\rho u^*{}^2}, \quad \tilde{T} = \frac{T - T_\infty}{T_s - T_\infty}, \quad Bo = \frac{g\beta(T_s - T_\infty)D^3}{\alpha^2}$$

$$Pr = \frac{\nu}{\alpha}, \quad \tilde{x} = \frac{x'}{D}, \quad \tilde{y} = \frac{y'}{D}, \quad \tilde{u} = \frac{u}{u^*}, \quad \tilde{v} = \frac{v}{u^*}$$

4.2. The boundary conditions

(a) Inlet

$$\tilde{v} = \frac{\partial \tilde{u}}{\partial \tilde{x}} = \tilde{T} = 0 \quad (24)$$

(b) Outlet

$$\tilde{v} = \frac{\partial \tilde{u}}{\partial \tilde{x}} = \frac{\partial \tilde{T}}{\partial \tilde{x}} = 0 \quad (25)$$

(c) Confining wall

$$\tilde{u} = \tilde{v} = \frac{\partial \tilde{T}}{\partial \tilde{y}} = 0 \quad 26$$

(d) Symmetry line

$$\tilde{v} = \frac{\partial \tilde{u}}{\partial \tilde{y}} = \frac{\partial \tilde{T}}{\partial \tilde{y}} = 0 \quad (27)$$

(e) On the cylinder

$$\tilde{u} = \tilde{v} = 0 \quad (28)$$

$$\tilde{T} = 1 \quad (29)$$

Problem was solved for different wall distance to cylinder diameter ratios ($t/D = 1.5, 2, 3$ and 6) and for different Rayleigh numbers ($Ra = 649, 767, 842, 910$ and 1000), but, for one wall height to cylinder diameter ratio ($H/D = 7$), using the CosMos finite element computer code.

Table 1 shows a comparison between the reported Nusselt numbers [5] and those calculated in the present

Table 1

Numerically calculated Nusselt number for $H/D = 7$, $Ra = 1000$ and $t/D = 1.5, 3$ and 6

t/D	Nu	
	Present investigation	Reported values [5]
1.5	3.7	3.4
3	3.9	3.9
6	3.5	3.4

investigation, for $Ra = 1000$. Agreement of the results is reasonable.

The number of nodes and elements used for each case are as given in Table 2.

5. Experimental solution

To verify validity of the numerical results, the same problem was also considered, experimentally.

Sets of data were generated for $Ra \approx 650, 768, 843$ and 910 and for the ratios $t/D = 1.5, 3, 6, 8, 12$ and ∞ . The H/D ratio was kept constant, equal to 7 .

5.1. Experimental setup and measurements

The experimental setup includes a single aluminum cylinder tube with 400 mm length and 6 and 5.5 mm external and internal diameters, respectively. This tube was heated, from inside, by a small electrical resistance element. Both ends of the cylinder were kept insulated by end caps. This helped in keeping the temperature uniform along the cylinder.

This cylinder was mounted on a frame made from Plexiglas. The confining walls were made from wood, covered with aluminum foil.

Temperature sensors LM35, with 0.5°C precision, were used to measure the cylinder's surface temperature, the wall temperature and the ambient temperature. The cylinder's surface temperature was controlled and kept constant at specified values by measuring the temperatures at three different axial locations and averaging the measured values. The surface tempera-

Table 2

The number of nodes and elements used in each case

t/D	1.5	2	3	6
Number of nodes	2009	1479	3144	1971
Number of elements	1846	1357	2890	1875

ture measuring positions were in the middle and at two different points, 180 mm away, to the left and to the right of the middle point.

Heat generated in the cylinder dissipated from its surface either by convection or by radiation. Neglecting the end losses, we can write;

$$Q_{\text{elec}} = Q_{\text{conv}} + Q_{\text{rad}} \tag{30}$$

where Q_{conv} and Q_{rad} are heat transfer from the surface of the cylinder by convection and radiation, respectively.

$$Q_{\text{conv}} = hA(T_s - T_\infty) \tag{31}$$

and after some manipulation it can be shown that [14],

$$Q_{\text{rad}} = A\epsilon\sigma \left[(T_s^4 - T_\infty^4) - 2F(T_w^4 - T_\infty^4) \right] \tag{32}$$

Therefore, the Nusselt number is defined as

$$Nu = \frac{hD}{k} = \frac{Q_{\text{elec}} - \pi DL\epsilon\sigma \left[(T_s^4 - T_\infty^4) - 2F(T_w^4 - T_\infty^4) \right]}{\pi kL(T_s - T_\infty)} \tag{33}$$

where F is the radiation shape factor [13].

Values of the Nusselt number, calculated by using the experimental results are presented in Table 3. Results in the last row of the Table 3, for $t/D = \infty$, is in a good agreement with values predicted by Eq. (11) [13], with maximum deviation of 5%.

6. Results and discussion

Sets of numerical and experimental data are generated and the Nusselt number is calculated for different wall spacing to cylinder diameter ratios and for different Rayleigh numbers. The wall height to cylinder diameter ratio is kept constant at $H/D = 7$. Some of the Nusselt numbers are presented in Tables 1 and 3. The

Table 3
The Nusselt numbers calculated using the experimental data

t/D	$Ra = 649.8$	$Ra = 767.7$	$Ra = 842.7$	$Ra = 910$
1.5	2.44	2.91	3.12	3.63
3	3.08	3.42	3.67	4.15
6	3.05	3.25	3.51	3.85
8	3.03	3.16	3.45	3.66
12	2.94	3.04	3.27	3.32
∞	2.87	2.96	3.02	3.08

Nusselt numbers are consistent with the reported values in the literature.

Results of the analytical investigation are used as a guideline for developing a general equation which gives the variation of the Nusselt number with t/D and Ra for the constant value of H/D .

The equation which fits the data well, is given as

$$\overline{Nu} = \frac{1.251}{\eta^{1/2}} \left(1 - e^{-1.236 \times 10^{-4} \eta^{3.5}} \right) + 0.75 \tag{34}$$

where

$$\overline{Nu} = \frac{Nu}{Ra_D^{0.188}} \tag{35a}$$

and

$$\eta = \frac{t}{D} Ra_D^{0.27} \tag{35b}$$

For the limits of $t/D \rightarrow 1$ and $t/D \rightarrow \infty$, Eq. (34) is simplified as

$$Nu \approx Ra_D (t/D)^3 \tag{36}$$

and

$$Nu \approx Ra_D^{0.188} \tag{37}$$

respectively, which are very similar to what was concluded from the analytical solution, Eqs. (21) and (11).

Eq. (34) is presented in Fig. 2, graphically, along with the experimental and numerical data. Agreement between the calculated and the predicted Nusselt numbers is good.

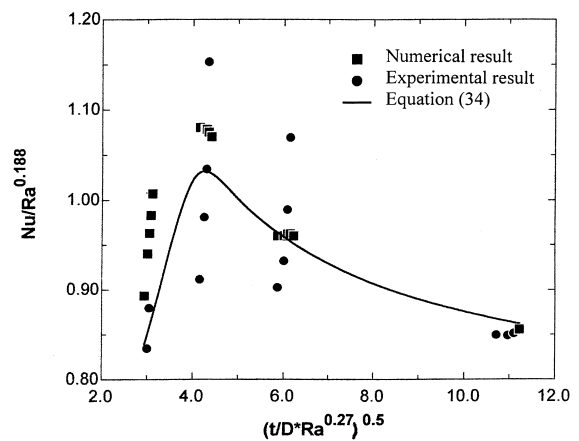


Fig. 2. A comparison between the Nusselt number predicted by the correlation (34) and the values obtained from the numerical and/or experimental data.

7. Conclusion

Effects of adiabatic confining walls on the free convection from a horizontal isothermal cylinder are studied analytically, experimentally and numerically. For the low Rayleigh numbers considered, an optimum wall to wall spacing was obtained for the maximum rate of heat transfer from the cylinder.

The analytical solution which is for the limits of $t/D \rightarrow 1$ and $t/D \rightarrow \infty$ are used as a guideline to incorporate the experimental and numerical results into a single correlation for the average Nusselt number. This correlation gives the variation of the Nusselt number with the Rayleigh number and wall to wall spacing to cylinder diameter ratio. The height of the walls is kept constant. The Nusselt numbers predicted from the correlation agree well with those calculated from the numerical and/or the experimental results.

Acknowledgements

The authors wish to acknowledge support for this research from the office of chancellor for research, Sharif University of Technology, Tehran, Iran.

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